A new mathematical formulation to integrate supply and demand within a choice-based optimization framework

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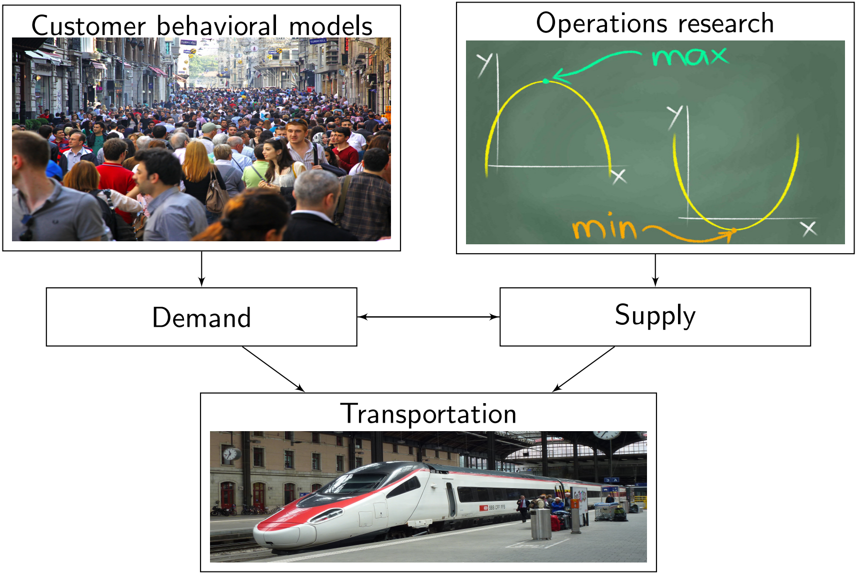
April, 2016

Outline

Introduction



Demand and supply

Customer behavioral models

- Given the configuration of the system ⇒ predict the demand
- Maximize satisfaction
- **Here:** discrete choice models

Operations Research

- Given the demand ⇒ configure the system
- Minimize costs
- **Here:** MILP

Discrete choice models in optimization problems

- Integrated choice model ⇒ source of nonconvexity
- Many techniques to convexify and linearize. **Here:** different approach
  - Nonconvex representation of choice probabilities
  - Include a wide class of discrete choice models

Utilities

Demand and supply

- Population of N individuals
- Set of products $\mathcal{C}$ in the market
  - artificial "opt-out" product
- $\mathcal{C}_n \subseteq \mathcal{C}$ subset of available products to individual n

Utility

U_{in} associated score to alternative i by individual n: $U_{in} = V_{in} + \varepsilon_{in}$

- V_{in}: deterministic part
- ε_{in}: error term

Behavioral assumption: n chooses i if U_{in} is the highest in $\mathcal{C}_n$

Probabilistic model

Choice

$$w_{in} = \begin{cases} 1 & \text{if } n \text{ chooses } i \\ 0 & \text{otherwise} \end{cases}$$

$\forall n, \forall i \in \mathcal{C}$

Availability

$$y_{in} = \begin{cases} 1 & \text{if } i \in \mathcal{C}_n \\ 0 & \text{otherwise} \end{cases}$$

$\forall n, \forall i \in \mathcal{C}$

$$w_{in} = 1 \Leftrightarrow y_{in} = 1 \text{ and } U_{in} \geq U_{jn}, \forall j \in \mathcal{C}_n$$

Probabilistic model

- $\Pr(w_{in} = 1) = \Pr(U_{in} \geq U_{jn}, \forall j \in \mathcal{C}_n)$
- $D_i = \sum_{n=1}^{N} \Pr(w_{in} = 1)$

Simulation

Non linearity

- D_i is in general non linear
- **Example:** $\Pr(w_{in} = 1) = \frac{y_{in} e^{V_{in}}}{\sum_{j \in \mathcal{C}} y_{jn} e^{V_{jn}}}$ (logit model)

Simulation

- Assume a distribution for ε_{in}
- Generate R draws $\xi_{in1} \ldots \xi_{inR}$
- r behavioral scenario
- The choice problem becomes **deterministic**

Demand model

$$U_{inr} = V_{in} + \xi_{inr} = \sum_k \beta_k x_{ink} + f(z_{in}) + \xi_{inr} \tag{1}$$

$\Rightarrow U_{inr}$ is not a random variable

Endogenous part of V_{in}

- Linear in the variables x_{ink}
- Decision variables (involved in the optimization problem)
- Assumption for the integration in a MILP

Exogenous part of V_{in}

- Depends on other variables z_{in}
- f not necessarily linear

Availability of alternatives (I)

Variables

- y_{in} decision of the operator

$$y_{in} = 0 \qquad \forall i \notin \mathcal{C}_n, n \tag{2}$$

- y_{inr} availability at scenario level (*e.g.* demand exceeding capacity)

$$y_{inr} \leq y_{in} \qquad \forall i, n, r \tag{3}$$

Idea: linearization of $U_{inr} y_{inr}$

$$\nu_{inr} = \begin{cases} U_{inr} & \text{if } y_{inr} = 1 \\ l_{inr} & \text{if } y_{inr} = 0 \end{cases}$$

Where $l_{inr} = \min\{U_{inr}\}$
$m_{inr} = \max\{U_{inr}\}$

Availability of alternatives (II)

Constraints

$$
\begin{aligned}
l_{inr} &\leq \nu_{inr}, & \forall i, n, r \quad (4)\\
\nu_{inr} &\leq l_{inr} + (m_{inr} - l_{inr}) y_{inr}, & \forall i, n, r \quad (5)\\
U_{inr} + (l_{inr} - m_{inr})(1 - y_{inr}) &\leq \nu_{inr}, & \forall i, n, r \quad (6)\\
\nu_{inr} &\leq U_{inr} & \forall i, n, r \quad (7)
\end{aligned}
$$

- $y_{inr} = 1 \Rightarrow$ Binding constraints: (6) and (7) $\Rightarrow \nu_{inr} = U_{inr}$
- $y_{inr} = 0 \Rightarrow$ Binding constraints: (4) and (5) $\Rightarrow \nu_{inr} = l_{inr}$

Highest utility among the available alternatives

Linearization of the maximum of variables

$U_{nr} = \max_{j \in \mathcal{C}_n} \{U_{jnr}\}$

Highest utility for individual n in scenario r: $\mu_{inr} = \begin{cases} 1 & \text{if } U_{nr} = U_{inr} \\ 0 & \text{otherwise} \end{cases}$

$$\nu_{inr} \leq U_{nr} \qquad \forall i, n, r \qquad (8)$$

$$U_{nr} \leq \nu_{inr} + M_{inr}(1 - \mu_{inr}) \qquad \forall i, n, r \qquad (9)$$

$$\sum_{i \in \mathcal{C}} \mu_{inr} = 1 \qquad \forall n, r \qquad (10)$$

where $M_{inr} = \max_{j \in \mathcal{C}} m_{jnr} - l_{inr}$

- $\mu_{inr} = 1 \Rightarrow U_{nr} = \nu_{inr} = U_{inr}$
- $\mu_{inr} = 0 \Rightarrow \nu_{nr} = l_{inr}$

Choice and availability

Constraints

$$\mu_{inr} \leq y_{inr} \qquad \forall i, n, r \qquad (11)$$

$$w_{inr} \leq \mu_{inr} \qquad \forall i, n, r \qquad (12)$$

$$w_{inr} \leq y_{inr} \qquad \forall i, n, r \qquad (13)$$

$$\sum_{i \in \mathcal{C}} w_{inr} = 1 \qquad \forall n, r \qquad (14)$$

- (11) An unavailable alternative cannot be the one with highest utility
- (12) An alternative without the highest utility cannot be chosen
- (13) An unavailable alternative cannot be chosen
- (14) Only one alternative is chosen

Modeling framework

Model (1)-(14)

- Linear in the variables
  - Any variable appearing linearly in U_{inr}
  - The availability variables y_{in}, y_{inr} and ν_{inr}
  - The preference variables μ_{inr}
  - The choice variables w_{inr}
- Demand within the market

$$D_i = \frac{1}{R} \sum_{n=1}^{N} \sum_{r=1}^{R} w_{inr}$$

- Further specifications
  - Capacity?
  - Price?

Maximization of revenues

Application

- Operator selling services to a market, each offered service:
  - Price
  - Capacity (number of customers)
- Demand is price elastic and heterogenous
- **Goal:** best strategy in terms of capacity allocation and pricing

Revenues

- p_{in} price that individual n has to pay to access service i

$$R_i = \frac{1}{R} \sum_{n=1}^{N} p_{in} \sum_{r=1}^{R} w_{inr}$$

- p_{in} endogenous variable $\Rightarrow$ R_i non linear

Pricing (I)

Linearization of R_i

- Discretization of the price $\Rightarrow p_{in}^1, \dots, p_{in}^{L_{in}}$
- Binary variables λ_{inl} such that $p_{in} = \sum_{l=1}^{L_{in}} \lambda_{inl} p_{in}^l$ and

$$\sum_{l=1}^{L_{in}} \lambda_{inl} = 1 \qquad \forall i, n \qquad (15)$$

- Revenues for alternative i

$$R_i = \frac{1}{R} \sum_{n=1}^{N} \sum_{l=1}^{L_{in}} \lambda_{inl} p_{in}^l \sum_{r=1}^{R} w_{inr}$$

- Still non linear $\Rightarrow \alpha_{inrl} = \lambda_{inl} w_{inr}$ to linearize it

Pricing (II)

Constraints

$$\lambda_{inl} + w_{inr} \leq 1 + \alpha_{inrl} \qquad \forall i, n, r, l \qquad (16)$$

$$\alpha_{inrl} \leq \lambda_{inl} \qquad \forall i, n, r, l \qquad (17)$$

$$\alpha_{inrl} \leq w_{inr} \qquad \forall i, n, r, l \qquad (18)$$

Objective function

$$\max R_i = \max \frac{1}{R} \sum_{n=1}^{N} \sum_{l=1}^{L_{in}} \alpha_{inrl} p_{in}^{l}$$

Capacity (I)

Priority list

- Who has access?
- We assume a priority list

$$y_{inr} \geq y_{i(n+1)r} \qquad \forall i, n, r \qquad (19)$$

Capacity

- c_i capacity of service i
- $c_{max} = \max_i c_i$, $c_{min} = \min_i c_i$
- $K_n = \max(n, c_{max})$

Constraints (I)

$$c_i(1 - y_{inr}) \leq \sum_{m=1}^{n-1} w_{imr} + (1 - y_{in})c_{max} \qquad \forall i, n, r \qquad (20)$$

- $y_{inr} = 0$ and $y_{in} = 1 \Rightarrow c_i \leq \sum_{m=1}^{n-1}$ (capacity is reached)
- $y_{inr} = y_{in} = 1$ and $y_{inr} = y_{in} = 0 \Rightarrow$ always verified

Capacity (II)

Constraints (II)

$$\sum_{m=1}^{n-1} w_{imr} + (1 - y_{in})c_{max} = (c_i - 1)y_{inr} + K_n(1 - y_{inr}) \quad \forall i, n \leq c_{min}, r \tag{21}$$

- $y_{inr} = y_{in} = 1 \Rightarrow 1 + \sum_{m=1}^{n-1} w_{imr} \leq c_i$
  (capacity must not be exceeded by the individuals choosing $i + n$)
- $y_{inr} = y_{in} = 0$ and $y_{inr} = 0, y_{in} = 1 \Rightarrow$ always verified

Parking choices

Original experiment

- [Ibeas et al., 2014] *Modelling parking choices considering user heterogeneity*
- Stated preferences survey (197 respondents)
- Analyze viability of an underground car park
- 8 scenarios suggested

Free on-Street Parking (FSP)

Free

Paid on-Street Parking (PSP)

Price levels: 0.6 and 0.8

Paid Underground Parking (PUP)

Price levels: 0.8 and 1.5

Choice model and preliminary experiments

Mixed Logit model

- **Attributes:** time to reach the destination
- **Socioeconomic characteristics:** residence, age of the vehicle
- **Interactions:** price and low income, price and residence
- **Random parameters:** access time and price

Preliminary experiment

- Subset of individuals
- Fixed capacity for the 3 alternatives

Results

N	R	cap FSP	cap PSP	cap PUP	Comp Time (s)	Obj
25	1	10	10	10	0.20	18.30
25	5	10	10	10	3.20	18.58
25	10	10	10	10	8.49	18.86
25	50	10	10	10	74.21	18.89
25	100	10	10	10	431.46	18.92
50	1	20	20	20	0.43	33.10
50	5	20	20	20	11.58	32.26
50	10	20	20	20	97.12	31.56
50	25	20	20	20	763.37	32.23
50	50	20	20	20	8744.14	31.60

Conclusions and future work

Conclusions

- High dimensionality of the problem
- Any assumption can be made for the ε_{in}

Future work

- Design of scenarios $\Rightarrow$ more experiments!
- Speed up the computational results
  - Preprocessing in particular cases (*e.g.* dominant alternatives)
  - Decomposition techniques (*e.g.* by scenario)
- Introduce new features (*e.g.* N as a group of individuals)

Questions?

A. Ibeas, L. dellOlio, M. Bordagaray, and J. de D. Ortzar. Modelling parking choices considering user heterogeneity. *Transportation Research Part A: Policy and Practice*, 70:41 – 49, 2014. ISSN 0965-8564. doi: http://dx.doi.org/10.1016/j.tra.2014.10.001. URL http://www.sciencedirect.com/science/article/pii/S0965856414002341.